The Locker Problem

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Oregon CIM Standard-Calculations and Estimation

Understand meanings of operations and how they relate to one another *(ODE Standard '06-'07)*.

Implement with Algebra 1, Geometry or Algebra 2 (at beginning of term to build problem solving skills) For Algebra 1-use with chapter on integers/number sense

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Task 1:

At Lovemath High School, there is a hallway that has 20 lockers in it, numbered 1 through 20. They are all closed while Mr. Iluv Schoo takes his class on a nature walk. When they return, they want to burn off some energy, and they make a plan:

The person from locker #1 ran down the hall and and opened every locker door. Then the person from locker #2 closed every second door, starting with locker #2. The person from locker #3 changed the state of every third locker starting with locker #3 (if it is open, close it, or if it is closed, open it). The person from locker #4 changed the state of every fourth locker, starting with locker #4. This continues until all of the 20 lockers have been passed by all 20 students. Which lockers are still open after the 20^{th} student passes through?

Task 2:

A new high school has just opened with an enrollment of 1000 students. The janitor closed all of the lockers and put a new coat of paint on the doors which are numbered 1 through 1000. The first student enters the school and opens all the lockers. The second student follows and closes every second locker starting with the second locker. The third student enters the school and reverses every third locker beginning with locker #3. This procedure continues until all 1000 students have passed by all the lockers. When they finish which lockers will be open?

- Part A: Which locker or lockers changed the most?
- Part B: What if we say there are "*n*" number of lockers in a school. If we did this procedure, how would we know which lockers would be open at the end? Write a rule that we could use.

Goal: Explain your thinking from part B on a poster showing your thinking, process and ideas.

Problem adapted from Fostering Algebraic Thinking, Mark Driscoll, 1999

Solution:

Task 1-The students should end up with 1, 4, 9, and 16.

Task 2-The students should end up with perfect square numbers. 1, 4, 9, 16, 25, 36, 49, 64, etc…
The number of times a locker changes is the same as the number of factors the number has. For example, the number 12 has factors 1, 2, 3, 4, 6, 12, so it changes six times. It would have changed an even number of times, so it ends up in the state it started in, closed. This is true for all lockers that have an even number of factors. The lockers that are left open have an odd number of factors. Ex: 9 Factors are 1, 3, 9 so it has changed an odd number of times, and is open.